Measurement-Based Probabilistic Timing Analysis for Multi-path Programs

L. Cucu-Grosjean\textsuperscript{4}, L. Santinelli\textsuperscript{4}, M. Houston\textsuperscript{2}, C. Lo\textsuperscript{4}, T. Vardanega\textsuperscript{3}, L. Kosmidis\textsuperscript{1}, J. Abella\textsuperscript{1}, E. Mezzetti\textsuperscript{3}, E. Quinones\textsuperscript{1}, F. J. Cazorla\textsuperscript{1}

24th Euromicro Conference on Real-Time Systems (ECRTS12)

The research leading to these results has received funding from the European Community’s Seventh Framework Programme [FP7/2007-2013] under grant agreement n° 249100.
Motivation

- **Critical Real-Time Embedded (CRTE) systems**
  - Used in Space, Aerospace, Transportation,… industries

- **CRTES requirements**
  - Recurring: Reduced development and production costs
  - Emerging: Increased functional value

- **More functional value → more computational power**
  - More complex SW
  - More complex HW: multicore and caches

- **Complex HW and SW affect time analysability**

- **PROARTIS view:**
  - Simple processors hard to analyse with current analysis techniques
  - Current analysis techniques cannot keep pace with novel HW
Execution history (EH)

- **Current architectures exploit knowledge on EH**
  - To improve average-case execution time (ACET)
  - Caches: Temporal and spatial locality
- The **ACET and WCET of programs heavily depend on EH**
- **Current analysis techniques require info about EH**
- **Limitations & increasing effort in acquiring EH info**
  - Complex processor architectures (IP protected)
  - Incomplete and/or inaccurate documentation
  - Program information may be unknown at analysis time
- **Reduction of available knowledge about HW/SW → pessimistic assumptions → degradation of the tightness of the WCET**
Execution history (EH)

- Current architectures exploit knowledge on EH
  - To improve average-case execution time (ACET)
  - Caches: Temporal and spatial locality
- The ACET and WCET of programs heavily depend on EH

- Design HW and SW whose execution time behaviour does not depend on execution history
  - Without removing performance-improving hardware!
  - The functional behaviour is left unchanged
- Provide new probabilistic timing analysis techniques

pessimistic assumptions \(\Rightarrow\)
degradation of the tightness of the WCET
**Probabilistically Analysable Real-Time Systems**

- Probability of appearance of an event *is not equal to its frequency of appearance within a finite time interval*

**Probability of 1, 1, 1, 1, 1?**

1, 4, 5, 2,… unknown

\[
\left( \frac{1}{6} \right)^5
\]
Probabilistic nature of the system

- pWCET estimations for high integrity systems
  - Counter intuitive
- Probabilistic modeling is in close match with actual nature of the system
  - Mechanical parts $\rightarrow$ designed with a probability of failure in mind
  - Hardware $\rightarrow$ affected by radiation, temperature, ...
    - Probability of failure
  - Failure rate for airborne applications $\rightarrow$ $10^{-9}$ per hour of operation
- The system as a whole has a distinct prob. of failure
- PTA: upperbounds the execution time of programs with an attached probability of exceedance
  - Time Failures can be considered just another type of failure
Probabilistic nature of the system

• pWCET estimations for high integrity systems
  ➢ Counter intuitive

• Probabilistic modeling is in close match with actual nature


  WWW.PROARTIS-PROJECT.EU

attached probability of exceedance

  ▪ Time Failures can be considered just another type of failure
Outline

• *Probabilistic Worst-Case Execution Time*
  ➢ Definition
  ➢ Extreme Value Theory
• *Measurement-based approach for single-path programs*
• *Measurement-based approach for multi-path programs*
• *Experiments*
• *Conclusions and ongoing work*
Probabilistic Timing Analysis

- PTA allows cutting the WCET bound tail at the level of probability suited for the system (e.g. $10^{-16}$ per hour of operation)
  - Probability of failure of the program $\times$ execution rate per hour

![Diagram showing observed execution times, WCET estimation with STA, and PTA over time with probability]

- WCET estimation with STA
- PTA
- Time
- Probability
What is Extreme Value Theory?

- **Measurement Based Probabilistic Timing Analysis**
- **The pWCET estimate is obtained by applying Extreme Value Theory**

Let $\{\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n\}$ be a sequence of i.i.d. random variables and let $M_n = \max\{\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n\}$. If $F$ is a non degenerate distribution function and there exists a sequence of pairs of real numbers $(a_n, b_n)$ such that $a_n \geq 0$ and $\lim_{n \to \infty} P\left( \frac{M_n - b_n}{a_n} \leq x \right) = F(x)$, then $F$ belongs to either the Gumbel, the Frechet or the Weibull family.
I.i.d. random variables

- **Independent random variables**
  - Random variables that describe events which are not related
  - “Event A: my laptop died” and “Event B: the beamer does not like my laptop”

- **Identically distributed random variables**
  - Random variables that have the same distribution function
  - “Event A: arrival of a client in a bank” and “Event B: arrival of a car at a gas station”

- **Both properties checked through statistical tests**
Required properties: identical distribution

• Data must be identically distributed
  - Hypothesis testing with Kolmogorov-Smirnov test
    - Two subsets of data randomly obtained from the trace
    - Distributions are compared looking at the maximum distance $P$ between the data sets
    - If $P$ is above a given threshold, data are identically distributed
      - Threshold (0.05) is chosen based on experience/common practice
Applying Extreme Value Theory

- **Measurement Based Probabilistic Timing Analysis**
- **First sound utilisation of EVT to pWCET estimating**

  - The hypothesis of independence and identical distribution is checked before any EVT utilisation
  - Continuous function: block maxima applied with a proper definition of minimum number of runs

  - Utilisation of a (proven) correct statistical test to check that data belong to the Gumbel domain
Required properties: Gumbel

- **Data must fit a Gumbel distribution**
  - Hypothesis testing with *exponential tail* test
    - A parameter \( P \) is obtained for the actual data
    - A confidence interval \( CI \) is obtained for the actual data assuming it fits a Gumbel distribution
    - If \( P \) belongs to \( CI \) then the data fit a Gumbel distribution

Execution times:

\[
\begin{align*}
6372 \\
3728 \\
6321 \\
8328 \\
8231 \\
9827 \\
\vdots
\end{align*}
\]

\( + \) Gumbel

Confidence interval \( CI \)

Parameter \( P \)
Steps of applying EVT (single-path programs)

- Observations
- Grouping
- Fitting
- Comparison
- Tail extension

• **Convergence**
  - Continuous rank probability score

\[
CRPS = \sum_{i=0}^{+\infty} [f_x(i) - f_y(i)]^2
\]

- Gumbel Fitting (ET)
- I.i.d tests

CRPS
What about multi-path programs?

- **Measurement Based Probabilistic Timing Analysis**
- **Multi-path programs**
  - Independent and identical distributions
    - Identical distributions ensured by the convergence process
  - Minimum number of observations
    - Each path must be observed a minimum number of times and it is ensured by the convergence process
  - Path coverage
    - pWCET estimate is obtained for the observed paths
Experiments /1

- All experiments run on a modified version of the SoCLib processor simulator
- Benchmarks
  - Eight of the EEMBC Autobench (single-path programs)
  - Seven of the Mälardarlen benchmarks (multi-path programs)
  - Our synthetic benchmark for which we know
    - Different paths of the program
    - Different input set that exercise each path
    - The longest path
SPTA*

- Static Probabilistic Timing Analysis
- The pWCET estimate is obtained by convolving the random variables describing the execution time of each instruction

\[
\begin{pmatrix}
1 & 2 \\
0.7 & 0.3 \\
\end{pmatrix}
\otimes
\begin{pmatrix}
7 \\
1 \\
\end{pmatrix}
=
\begin{pmatrix}
1+7 & 2+7 \\
1 \cdot 0.7 & 1 \cdot 0.3 \\
\end{pmatrix}
=
\begin{pmatrix}
8 & 9 \\
0.7 & 0.3 \\
\end{pmatrix}
\]

*ACM TECS 2012 – « Probabilistically analysable real-time systems»
Experiments /2 (single-path programs)

Aifirf (EEMBC)

Minimum number of runs

<table>
<thead>
<tr>
<th>Probability</th>
<th>AI</th>
<th>AT</th>
<th>CA</th>
<th>CN</th>
<th>PU</th>
<th>RS</th>
<th>TB</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-13}$</td>
<td>9%</td>
<td>5%</td>
<td>6%</td>
<td>5%</td>
<td>5%</td>
<td>2%</td>
<td>2%</td>
<td>6%</td>
</tr>
<tr>
<td>$10^{-16}$</td>
<td>15%</td>
<td>7%</td>
<td>8%</td>
<td>7%</td>
<td>7%</td>
<td>3%</td>
<td>3%</td>
<td>9%</td>
</tr>
</tbody>
</table>

MBPTA versus wrt SPTA
Experiments /3 (multi-path programs)

### Synthetic benchmark

![Graph showing comparison between SPTA wc and multiple mpEVT programs](image)

**MBPTA versus SPTA (Mälardarlen benchmark)**

<table>
<thead>
<tr>
<th>probability</th>
<th>BS</th>
<th>CNT</th>
<th>COM</th>
<th>CRC</th>
<th>INS</th>
<th>QSO</th>
<th>SEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-13}$</td>
<td>7%</td>
<td>2%</td>
<td>4%</td>
<td>1%</td>
<td>9%</td>
<td>7%</td>
<td>5%</td>
</tr>
<tr>
<td>$10^{-16}$</td>
<td>8%</td>
<td>2%</td>
<td>5%</td>
<td>1%</td>
<td>11%</td>
<td>8%</td>
<td>6%</td>
</tr>
</tbody>
</table>
Conclusions and ongoing work

• Proven that MBPTA is a sound utilisation of EVT
  ➢ But value of pWCET bound is limited to the path coverage achieved in the observations
  ➢ Proof of the limited pessimism of EVT application

• Defining relevant test input data
• Proposing a hybrid probabilistic timing analysis
Measurement-Based Probabilistic Timing Analysis for Multi-path Programs

L. Cucu-Grosjean⁴, L. Santinelli⁴, M. Houston², C. Lo⁴, T. Vardanega³, L. Kosmidis¹, J. Abella¹, E. Mezzetti², E. Quinones¹, F. J. Cazorla¹

24th Euromicro Conference on Real-Time Systems (ECRTS12)

The research leading to these results has received funding from the European Community’s Seventh Framework Programme [FP7/2007-2013] under grant agreement nº 249100.