Schedulability Analysis of Periodic Tasks Implementing Synchronous Finite State Machines

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Outlines

• Motivations
• Synchronous FSMs
  – actions, not tasks
    • Need to map reactions into tasks
  – Applicability of Existing Task Models
• Schedulability Analysis Overview
• Efficient Calculation of RBF and DBF
  – Execution Request Matrix
  – Periodicity of Execution Request Matrix
• Summary and Future Work
Model-based Design

- Popular in many application domains of real-time systems
  - Automotive
  - Avionics
- To deal with complexity
  - Model everything for design (engineering) and analysis (science)
  - It is necessary to select a modeling language in the most natural and easy way
- The four tenets on the right are fundamental to model-based design
  - **No program by hand**
  - **Starting point is functional model**
  - **Automatic generation of implementation is key**
  - **Synthesis of tasks, priorities, allocation, communication mechanisms …**
Motivations

- Synchronous FSMs are used in the most popular model-based design tools
  - SCADE
  - Simulink/Stateflow

The system is a network of “dataflow” blocks and Stateflow (EFSM) blocks.
Synchronous Finite State Machines

- Event $e_i$:
  - Period $T_i$
  - Offset = 0
- State $S_i$
- Transition $S_i$-$\rightarrow$ $S_j$
  - Trigger event
  - Action $a_k$:
    - WCET $C_k$
  - guard, priority
- Hyperperiod $H = \text{lcm}$ of event periods
- Scheduled with static priority
  - As in commercial code generators (Simulink Coder, dSPACE TargetLink)
Existing Task Models

• Actions and tasks:
  – Assumption: all actions are executed by a single task
  – Other options are possible

• Digraph task model [1] and its extension [2]
  – Accuracy issue
    • Arbitrary offsets
    • Dynamic priority scheduling (EDF)
  – Efficiency issue
    • Patterns of trigger events repeat every hyperperiod
    • Further periodicity by max-plus algebra

Schedulability Analysis Overview

Schedulability Analysis for Task $i$

FOR each priority level-$i$ busy period $[s,f)$

IF $\exists t \in [s,f), \forall t' \in [s,t]$ such that

$$\tau_i \cdot dbf[s,t] + \sum_{j \in h_p(i)} \tau_j \cdot rbf[s,t'] > t' - s$$

THEN return unschedulable

ENDIF

Return schedulable

Remaining Question:
how to efficiently calculate $rbf(\Delta)$ and $dbf(\Delta)$ for a given time interval $\Delta$?
Event Sequence Pattern and Reachability Graph

Need to compute the request bound function

![Event sequence in one hyperperiod]

![Reachability graph in one hyperperiod]
Refinement of rbf

• $rbf_{i,j}(\Delta)$:
  – source state of the first transition is $S_i$
  – sink state of the last transition is $S_j$

• $rbf_{i,j}(\Delta)$ is **additive** (but $rbf(\Delta)$ is not)
  – $rbf_{i,j}[s, f] = \max_m (rbf_{i,m}[s, t] + rbf_{m,j}[t, f])$
  – $rbf_{i,j}[s, f]$ for a long interval $[s, f)$ can be computed from its values for shorter intervals $[s, t)$ and $[t, f)$

Key property to enable dynamic programming techniques
Execution Request Matrix

• The request bound function in one hyperperiod
  – \( X = (x_{i,j}) \), where \( x_{i,j} = rbf_{i,j}[0,H) \)

\[
\begin{bmatrix}
0.65 & 0.9 & 1.0 \\
0.45 & 0.7 & 0.8 \\
0.95 & 1.2 & 1.3 \\
\end{bmatrix}
\]

event sequence in one hyperperiod

\[
S1 \xrightarrow{e_1/a_1} S2 \xrightarrow{e_1/a_3} S3 \xrightarrow{e_2/a_2} S1 \xrightarrow{e_1/a_1} S2 \xrightarrow{e_1/a_3} S3
\]

\[
\Rightarrow x_{1,3} = 1.0
\]

\[
\Rightarrow X = \begin{bmatrix}
0.65 & 0.9 & 1.0 \\
0.45 & 0.7 & 0.8 \\
0.95 & 1.2 & 1.3 \\
\end{bmatrix}
\]
Execution Request Matrix

- The request bound function over several hyperperiods

\[ X^{(k)}(k+1) = \begin{bmatrix} 0.65 & 0.9 & 1.0 \\ 0.45 & 0.7 & 0.8 \\ 0.95 & 1.2 & 1.3 \end{bmatrix} + k \times 1.3 \]

- \( X^{(k)} = (x^{(k)}_{i,j}) \), where \( x^{(k)}_{i,j} = rbf_{i,j}[0,kH] \)

\[ \forall i, j, \forall 1 \leq l < k \quad x^{(k)}_{i,j} = \max \left( x^{(l)}_{i,m} + x^{(k-l)}_{m,j} \right) \]

This indicates some additional periodicity
Basics on Max-Plus Algebra

• Operations maximum (denoted by the max operator $\oplus$) and addition (denoted by the plus operator $\otimes$)
  \[ a \oplus b = \max(a, b) \quad a \otimes b = a + b \]

• Multiplication of two square matrices
  \[ A \otimes B = C, \text{ where} \]
  \[ c_{i,j} = \oplus \left( a_{i,m} \otimes b_{m,j} \right) = \max_m (a_{i,m} + b_{m,j}) \]
Periodicity of Matrix Power in Max-Plus Algebra

- Studied by its corresponding digraph $\mathcal{G}(X)$

\[
X = \begin{bmatrix}
1 & 2 & 3 \\
1 & 0.65 & 0.9 & 1.0 \\
2 & 0.45 & 0.7 & 0.8 \\
3 & 0.95 & 1.2 & 1.3 \\
\end{bmatrix}
\]

Edge $(i,j)$ has weight equal to matrix element $x_{i,j}$
Periodicity of Matrix Power in Max-Plus Algebra

• If the digraph $\mathcal{G}(X)$ is strongly connected
  – $X^{(k+p)} = X^{(k)} + p \times q$ for sufficiently large $k$
  – $p = \text{maximum cycle mean of } \mathcal{G}(X)$
  – $q = \text{lcm}$ of all the cycles with mean equal to $p$

\[ p=1.3, \ q=1 \implies X^{(k+1)} = \begin{bmatrix} 0.65 & 0.9 & 1.0 \\ 0.45 & 0.7 & 0.8 \\ 0.95 & 1.2 & 1.3 \end{bmatrix} + k \times 1.3 \]
The Efficient Way of Calculating \( rbf \)

- For small intervals

\[
\begin{align*}
rbf_{i,j}[s,f] &= \max_{k,l} (rbf_{i,k}[s,n_s H] + rbf_{k,l}[n_s H, n_f H]) \\
&\quad + rbf_{l,j}[n_f H, f]) \\
&= \max_{k,l} (rbf_{i,k}[s,n_s H] + x^{(n_f-n_s)}_{k,l} \\
&\quad + rbf_{l,j}[0, f - n_f H])
\end{align*}
\]

- For large intervals: the above equation applies, but

\[
x^{(n_f-n_s)}_{k,l} = x^{(n)}_{k,l} + (n_f - n_s - n) \times q_{k,l}(k)
\]

where \( n \leq d \) and \( n + p > d \), \( n_f - n_s \equiv n \equiv k \mod p \).

Asymptotic complexity independent from length of interval
Summary and Future Work

• Efficient and accurate schedulability analysis
  – Event sequence pattern within one hyperperiod
  – Max-plus algebra for evaluating the periodicity of the execution request matrix

• Multi-task implementation of an FSM
  – Issues with single task implementation
    • all actions executed at the same priority level
    • tight deadline (equal to the gcd of event periods)
    • inflexible for avoiding overhead from communication

• Extension of periodicity of $rbf$ and $dbf$ to generic digraph task models
Thank you!

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